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A hybrid non-dominated sorting genetic algorithm with local search for portfolio selection problem with cardinality constraints

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Abstract

The Cardinality-Constrained Portfolio Selection Problem (CCPSP) consists of allocating resources to a limited number of assets. In its classical form, it is represented as a multi-objective problem, which considers the expected return and the assumed risk in the portfolio. This problem is one of the most relevant subjects in finance and economics nowadays. In recent years, the consideration of cardinality constraints, which limit the number of assets in the portfolio, has received increased attention from researchers, mainly due to its importance in real-world decisions. In this context, this paper proposes a new hybrid heuristic approach, based on a Non-dominated Sorting Genetic Algorithm with Local Search structures, to solve PSP with cardinality constraints, aiming to overcome the challenge of achieving efficient solutions to the problem. The results demonstrated that the proposed algorithm achieved good quality results, outperforming other methods in the literature in several classic instances.

Keywords: portfolio selection problem, cardinality constraints, genetic algorithm, multiobjective optimization

Introduction

The Portfolio Selection Problem (PSP), which consists of the allocation of resources in a finite number of assets, is one of the most important topics regarding the financial and economic issues nowadays. As there is a trade-off to be surpassed (between risks and returns), the assets portfolio selection aims to decide in which assets one should invest and in which proportions, considering the available capital.

One of the first approaches based on Operational Research to treat this problem was proposed by Markowitz (1952), a model named Mean-Variance. The formulation consists of a problem with two non-linear objectives that aim to maximize the return and minimize the risk of the portfolio. This model can also be described as a problem of quadratic programming that minimizes the portfolio risk, and the smallest acceptable return is considered through a linear constraint.

The Mean-Variance Model was fundamental so that the thematic of asset portfolio optimization could be widespread and widely studied worldwide. Since Markowitz's seminal works, the subject is treated as an optimization problem with the two mentioned non-linear objectives: the expected return that can be measured by the mean return and must be maximized; and the risk of the portfolio, usually measured through the variance, that must be minimized (Markowitz, 1952; Sharpe, 1989).

As this sort of problem involves two objectives that cannot be simultaneously optimized, there is not only one optimal solution to be found, but also a set of efficient solutions, named Pareto border, in which a range of solutions is offered to the investors, making possible the proper decision-making, according to the risks one wishes to assume.

The approach proposed by Markowitz (1952) is based on the decision-making that will form a set of basis portfolios for the construction of Pareto border. The model is based on a simplistic market, without many factors associated with a real financial market, such as a minimum transaction lot. However, the simple consideration of cardinality constraints in the classical problem transforms it into a Mixed-Integer Quadratic Problem proved to be NP-Hard by Moral-Escudero, Ruiz-Torrubiano and Suárez (2006).

Many heuristic approaches have been used to solve portfolio optimization problems with constraints of the market's reality. The PSP considering the cardinality constraints and the investment limitation per asset (CCPSP), represents the most studied variation of the PSP in the literature, mainly due to the outstanding importance given to these constraints in the financial market decisions.

Chang, Meade and Sharaiha (2000) were the first authors to consider and report the results for the CCPSP in the literature. The authors used three methods to its resolution: Genetic Algorithm (GA), Tabu Search (TS), and Simulated Annealing (SA). The achieved results in the present instances of the OR-Library were shown as effective at that time. The work also allowed us to conclude that the existence of cardinality constraints implies in a non-continuous efficiency frontier.

Armananzas and Lozano (2005) used three approaches to treat the problem: Greedy Local Search (GLS), SA e Ant Colony Optimization (ACO), while Moral-Escudero, Ruiz-Torrubiano and Suárez (2006) implemented a Hybrid Genetic Algorithm (HGA) that combines genetic algorithm with quadratic programming.

Fernandéz and Goméz (2007) treated the CCPSP through a heuristic-based in neural networks (NN), called Hopfield network. Moreover, the authors implemented the heuristics proposed by Chang et al. (2000) and compared the results obtained in the instances of the OR-Library. The results indicated that none method was clearly superior to the others.

Cura (2009) implemented a Particle Swarm Optimization (PSO) for the resolution of the problem. The study compared the results to the ones achieved by the methods proposed by Chang et al. (2000). It was noted that none of the methods showed a clear superiority compared to the others.

Anagnostopoulos and Mamanis (2009) developed a GA based on NSGA. The authors used a method to solve instances of Athens Stock Exchange, and the achieved results were compared to a generic GA implemented. The results indicated that the developed method is more promising to this class of problems. Pai and Mitchel (2009) treated the problem through an approach based on MOEA, that uses a K-means clustering procedure to remove cardinality constraints and simplify the model.

Woodside-Oriakhi, Lucas and Beasley (2011) applied three algorithms and conducted a study with relation to the performance of its efficient frontiers, being them a GA, a TS, and a SA. They compared the results to the ones obtained by Chang et al. (2000), showing that their approaches reached competitive results.

Anagnostopoulos and Mamanis (2011) proposed five evolutionary algorithms to solve the problem, based on Niched Pareto Genetic Algorithm (NPGA), NSGA, Pareto Envelope-based Selection Algorithm (PESA), Strength Pareto Evolutionary Algorithm (SPEA), and E-multiobjective Evolutionary Algorithm (e-MOEA).

Mishra, Panda and Majhi (2014) used a MOEA approach based on Multiobjective Particle Swarm Optimization (MOPSO). The authors compared the results obtained to the other five MOEAs, one of them based on decomposition (MOEA/D), besides four mono objective algorithms: GA, SA, TS and PSO. The instances of the OR-Library were used, besides an additional one containing 500 assets. The results indicated that the method could find good quality solutions in a reasonable time, having the MOPSO achieved most of the best results.

Salahi, Daemi, Lofti and Jamalian (2014) proposed two heuristic approaches to solve the CCPSP, one based on PSO and the other in Harmony Search (HS). The instances of the OR-Library were tested, and the results indicated that the HS was significantly superior to the PSO, mainly in the higher instances. Following this same line, Sabar and Kendall (2014) implemented an algorithm based on HS to solve the problem, showing that the method can obtain quality solutions to the problem. While Baykasoğlu, Yunusoglu and Özsoydan (2015) applied for the first time an algorithm Greedy Randomized Adaptive Search Procedure (GRASP), comparing the obtained results to the ones achieved by Fernandéz and Goméz (2007).

Liagkouras and Metaxiotis (2018) implemented an MOEA-based method for the problem. The authors proposed efficient mechanisms and mutation strategies that allowed the method to achieve competitive results.

Kaucic (2019) applied a MOPSO to the PSP with cardinality and risk constraints. The study was one of the forerunners to consider a PSP with these characteristics and restrictions. Silva, Herthel and Subramanian (2019) developed a generalist approach, based on MOPSO, to treat a class of PSP problems, including CCPSP. The authors reported efficient results in several PSPs, presenting and comparing the results considering several metrics present in the literature.

Still considering a MOPSO-based approach, Zhao, Chen, Zhan and Kwong (2021) treated the problem through a modified MOPSO. The authors reported the results achieved, mainly through the hypervolume (HV) and inverted generation distance (IGD) metrics, presented with more details in the results section of this work. The results achieved by the authors demonstrated an efficient speed of convergence of the solution.

Kalaycy, Polat, and Akbay (2020) considered a hybrid metaheuristic for the problem. The method considers aspects present in genetic algorithm and particle swarm optimization approaches. In a later study, Akbay, Kalayci and Polat (2020) implemented a local search-based approach with quadratic programming. The developed approach allowed to achieve better results in the variance of return error metrics compared to other methods compared in the literature.

Xiong, Wang, Kou and Xu (2021) also developed a hybrid approach based on an evolutionary algorithm for the PSP with periodic reinvestments. The study was one of the first to address a variant of the problem with these characteristics. In this path of new proposed variants, Khodamoradi, Salahi and Najafi (2021) proposed a new variant of the CCPSP, which considers, in addition to cardinality restrictions, short selling and risk-neutral interest rate. Leung, Wang & Che (2022) implemented a neural network-based approach that achieved solid results on four real instances.

Besides the works mentioned above, many others studied the CCPSP (and variants) and proposed alternative approaches (Khan, Cao & Li, 2022; Rasoulzadeh, Edalatpanah, Fallah & Najafi, 2022; Golmakani & Fazel, 2011; Liagkouras & Metaxiotis, 2014; Deng, Lin & Lo, 2012; Chi, Cheng & Bai, 2014; Sadigh, Mokhtari, Iranpoor & Ghomi, 2012; Xu, Zhang, Liu & Huang, 2010).

It is essential to highlight that the interest in studying the topic of CCPSP and its variants has grown significantly in recent years. This fact occurs mainly due to the importance given by the market in adopting optimization approaches in its decision-making processes. Although many studies have proposed methods of different natures, most of them consider a limited set of performance metrics in evaluating their results, which makes it difficult to compare them with other proposed approaches. Moreover, it also compromises the evaluation of the solutions quality since other existing metrics allow evaluation under other important aspects.

In this context, this work aims to present a heuristic approach based on a Multiobjective Genetic Algorithm to resolve the CCPSP, named Adaptive Non-dominated Sorting Genetic Algorithm (ANSGA), with the objective of achieving more efficient solutions considering all the main existing metrics reported in the literature, making it possible to achieve better quality solutions at a lower computational cost.

Theoretical Background

Multiobjective Optimization

Multiobjective approaches attempt to simultaneously optimize multiple objective functions that are conflicting with each other. It is a problem in which there is a vector of decision variables that must not only satisfy a set of constraints of the problem but also optimize a set of objective functions of the problem, as can be seen in the Equations (1)-(3) (Mishra et al., 2014).

$$Min/Max \ \ f_m(\vec{x}), m = 1, \dots, M \tag{1}$$

Subject to:

$$g_j(\vec{x}) \geq 0, j = 1, \dots, J \tag{2}$$
$$x_i^I \leq x_i \leq x_i^S, i = 1, \dots, N \tag{3}$$

Where $\vec{x}$ is the set of decision variables of the problem, $\vec{x} = (x_1, x_2, \dots, x_N)^T$.

The search space is limited by $x_i^I \leq x_i \leq x_i^S, \forall i \ = \ 0, \dots, n$, where x_i^I and x_i^S respectively represent the inferior and superior limits that can be assumed by the variable x_i. N indicates the number of decision variables of the problem; M is the number of objective functions and j, the quantity of the further constraints of the problem.

A given solution vector $\vec{u} = (u_1, u_2, \dots, u_N)^T$ is said to be dominant of other $\vec{v} = (v_1, v_2, \dots, v_N)^T$ if, and only if:

$$f_i(\vec{u}) \succeq f_i(\vec{v}), \forall i = 1, \dots, M \tag{4}$$
$$f_i(\vec{u}) \succ f_i(\vec{v}), \exists i = 1, \dots, M \tag{5}$$

Where the relation $\succeq$ indicates that a solution is better or equal to another for a determined objective, while $\succ$ indicates that the solution is better. Thus, as the Equations 4 and 5 show, so that the solution $\vec{u}$ is dominant of the solution $\vec{v}$, the performance of the solution $\vec{u}$ must be better or equal to all the objective functions of the problem, having, yet, obtained in at least one of them, a better performance.

The further solutions that are not dominated by some other are called non-dominated solutions and represent a set that composes the Pareto border, representing the solutions for a multiobjective problem. The Pareto border represents a curve indicating all the non-dominated solutions obtained to the problem.

Mean-variance Portfolio Selection Problem

Proposed by Markowitz (1952), the model consists of a bi-objective optimization problem of two non-linear functions that represent the portfolio's return and risk, where the variance is used to measure the risk of the portfolio and should be minimized in the objective function (6). In contrast, the return expected of the portfolio is treated through the objective function (7) and should be maximized.

The constraints on the problem limit that the investments, both total in the portfolio (8) and individual in the assets (9), do not exceed 100% of the total amount that can be invested and do not be lower than 0.

$$z_1 = min \sum_{j \in A} \sum_{k \in A} w_j w_k \sigma_{jk} \quad (6)$$

$$z_2 = max \sum_{j \in A} w_j r_j \quad (7)$$

Subject to:

$$\sum_{j \in A} w_j = 1 \quad (8)$$

$$0 \leq w_j \leq 1, \qquad j \in A \quad (9)$$

Where A represents the number of available assets to compose the portfolio, σ_{jk} indicates the covariance between the assets j and K. w_j indicates the proportion of the investments in each asset, while r_j represents the mean return of the asset j.

A second approach to solve the classical problem followed by some studies consists of treating the expected return and the risk of the portfolio in the same function, weighting the objectives in the function of a parameter λ ϵ [0; 1], which indicates the aversion to risks of the investor. In this way, the lower the λ, the lower the aversion of investors to risks, while the larger the lambda, the lower will be the acceptable risk of the portfolio. The objective for the problem is shown below:

$$z = min\, \lambda \left[\sum_{j \in A} \sum_{k \in A} w_j w_k \sigma_{jk} \right] - (1 - \lambda) \sum_{j \in A} w_j r_j \qquad (10)$$

This procedure transforms the problem into a mono objective to minimize the risk of the portfolio and maximize the expected return according to the variation of the risk aversion parameter (λ), as the objective function (10) shows. The formulation is also constrained by equations (8) and (9) as the first formulation. With the variation of λ, many different trade-offs between returns and risks are obtained.

In this form of treatment of the problem, the Pareto border is formed from non-dominated solutions obtained by solving the problem for each variation of λ.

2.3 Cardinality Constrained Portfolio Selection Problem

The Cardinality Constrained Portfolio Selection Problem (CCPSP) consists of the same classical problem presented by Markowitz (1952) with the inclusion of cardinality constraints and investment limitations per asset.

This problem is one of the most commonly studied and treated in the literature of PSPs. This fact is mainly due to a practical application, as investors widely consider these restrictions. Another point that justifies this fact is the complexity of the problem, which motivates researchers to develop more efficient methods. The formulation below expresses the CCPSP.

$$f_1 = min \sum_{j \in A} \sum_{k \in A} w_j w_k \sigma_{jk} \qquad (13)$$

$$f_2 = max \sum_{j \in A} w_j r_j \qquad (14)$$

Subject to:

$$\sum_{j \in A} w_j = 1 \qquad (15)$$

$$\sum_{j \in A} z_j = K \qquad (16)$$

$$\epsilon_j z_j \leq w_j \leq \delta_j z_j, \qquad j \in A \qquad (17)$$

$$z_j \in \{0,1\}, \qquad j \in A \qquad (18)$$

Where z_j represents a binary variable that indicates 1, if the asset j is present in the portfolio, and 0, otherwise. K represents the quantity of assets that must composes the portfolio, while ϵ_j and δ_j represent, respectively, the lower and upper boundary of the proportion that can be invested in the asset j.

Despite most of the studies deal with cardinality, as the Equation (16), It can still be found in another way in the literature. Some works consider the cardinality constraint as being of inequality, through the lower (K_{min}) and superior upper (K_{max}) boundary (19).

$$K_{min} \leq \sum_{j \in A} z_j \leq K_{max} \tag{19}$$

An Adaptive Hybrid Non-dominated Sorting Genetic Algorithm

To solve the investment portfolio selection problem, a multi-objective evolutionary algorithm was proposed based on a genetic algorithm called Adaptive Hybrid Non-dominated Sorting Genetic Algorithm II (HNSGA). To achieve a higher speed of convergence towards more efficient Pareto boundaries, solutions ranking mechanisms were implemented in the iterative search process, based on structures proposed by Zhao, Liu, Zhang and Liu (2019) and Ma, Silva & Kuang (2019).

Hybrid approaches with similar characteristics have shown efficient results in several optimization problems in the literature, such as Facility Layout Problem (Huo, Liu & Gao, 2021), Traffic Signal Control (Nguyen, Passow & Yang, 2016), Deployment in Multi-cloud (Ma et al., 2019), Multi-docking Cross-docking (Guo, Chen & Ruan, 2012), Job Shop Scheduling (Erfani, Ebrahimnejad & Moosavi, 2020), and Batch Scheduling (Zhao, Liu & Zhou, 2020).

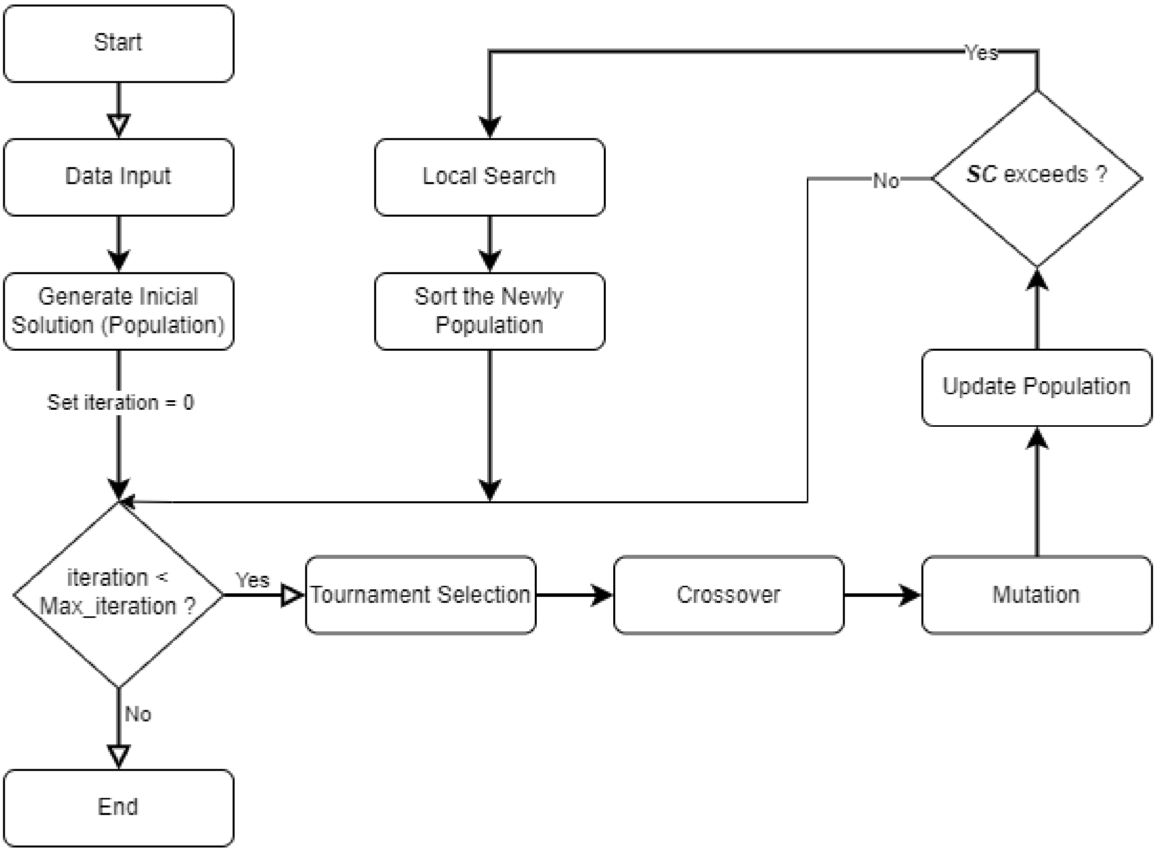
The NSGA-II, in its standard form, was proposed by Deb, Pratap, Agarwal and Meyarivan (2002) as a more efficient proposal for the application of a genetic algorithm in multi-objective problems. As a rule, the method initially randomly generates an initial population (P_0) of size N. Then, this population of individuals is categorized into hierarchical levels (boundaries) that represent dominance over each other, according to the existing goals. The first defined frontier is classified as non-dominated, while the individuals of the first frontier dominate the second and consecutively.

Each individual i is assigned a value referring to its ranking f_i (fitness), based on the level of the frontier to which it belongs. Thus, smaller fitness values indicate that individuals are closer to the efficient global frontier in minimization problems. Additionally, we considered a variable called crowding distance (CD_i) for each individual i. The selection (ranking of individuals) is performed based on their classification (fitness) and crowding distance, which is calculated only if the fitness of the individuals is equal. Such a mechanism is essential for the efficiency and convergence of the genetic algorithm.

After the selection procedure, the genetic algorithm will execute the basic structures predicted. The selected current population generates the children through crossover and mutation operations. Subsequently, an intermediate population composed of the current population and the offspring generated are classified again based on the non-dominance relationship between the individuals. The best individuals are selected, considering the measurement of the CD_i of each individual i, which will form the next generation of individuals in the next iteration. The flowchart in Figure 1 illustrates the reported steps.

Figure 1

Flowchart of HNSGA



Source: The Authors (2022).

Initialization

The initialization phase of the algorithm consists of randomly generating a population of individuals that represent solutions to the problem. During the greedy generation procedure, are adopted some feasibility mechanisms to ensure that the individuals generated respect the constraints of the problem, especially the maximum and minimum limit values of the proportions of investments in each asset, that is, ϵ_j and δ_j, as well as the existing cardinality constraint (K).

Algorithm 1 - Initialization

1: Initialization Procedure (K, ϵ_j, δ_j)
2: $ub =\ 1$ (Amount to be allocated)
3: **while** the cardinality constraint is not satisfied, **do**
4: Randomly select an asset j from the portfolio
5: Obtain w_j through a random procedure, respecting $\epsilon_j \leq w_j \leq \delta_j$ and $w_j \leq ub$
6: Update the remaining limit to be allocated: $ub = ub - w_j$
7: end **while**

Evaluation

The evaluation stage of the proposed algorithm consists of determining the suitability of the individuals in the population. In this way, is measure the performance for each objective of the problem (expected return and portfolio risk) for each individual of the population.

Subsequently, based on the evaluation performed, a procedure is performed to update the set of non-dominated solutions H. As a result, a new set of solutions will form, defining the Pareto frontier for the problem. After this procedure, at each iteration, the new individuals generated by the considered search engines are checked against the current set H. If any of them is not dominated by any individual present in H, the individual is added to the set H. Furthermore, if the individual added dominates some of the individuals already present in H, these individuals are discarded from the set, generating a new updated Pareto frontier.

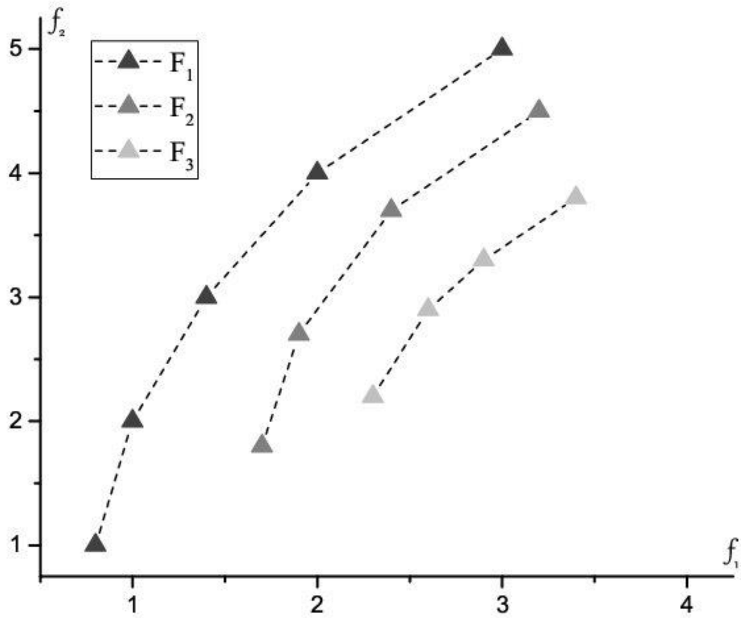
Non-dominated Sorting Crowding Distance Procedure

As previously highlighted, the non-dominated mechanism has been widely adopted in genetic algorithms to address multi-objective optimization problems. The ranking procedure is performed in two main steps: Non-dominated Sorting (NDS) and Crowding Distance Procedure (CD).

The first step consists of executing the NDS mechanism on the current population. The mechanism consists of, given a set of feasible solutions to the problem, building all possible Pareto boundaries. Subsequently, these frontiers are sorted into hierarchical levels based on the quality of the frontiers, i.e., the closer to the optimal frontier, the better. Figure 2 illustrates the procedure with three arbitrary frontiers.

Figure 2

Exemplification of Pareto Frontiers

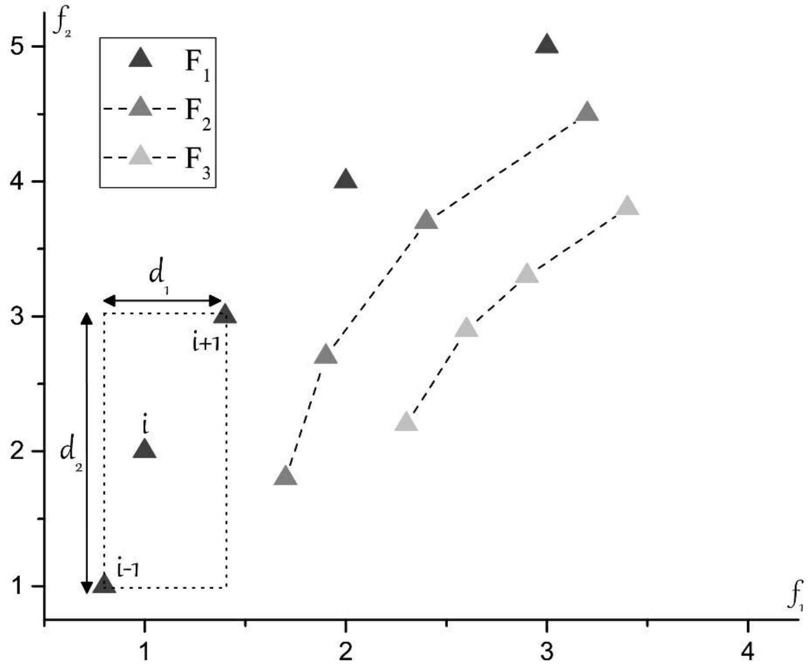


Source: Silva et al. (2019).

After performing the NDS, a CD procedure to verify the quality of the individuals (border points) is performed. The mechanism aims to calculate the area produced by the distance between the two nearest individuals for each individual i, i.e., $(i + 1)$ and $(i - 1)$. Thus, the mechanism allows estimating the density of individuals around a given individual on the frontier, as illustrated in Figure 3.

Figure 3

Crowding Distance Mechanism



Source: Silva et al. (2019)

In order to calculate each individual's CD_i, it is necessary to sort the individuals concerning their two objectives initially. For the individuals at the borders of the list sorted, infinity values are assigned to their CD_i since they do not have two neighboring individuals. Thus, the distances between the two individuals $(i+1)$ and $(i-1)$ are calculated, and subsequently, the CD_i of the individual i is measured. The Algorithm 2 illustrates the step-by-step procedure.

Algorithm 2 - Crowding Distance Mechanism

1: Input: H
2: **while** $m < m_{max}$ **do**
3: Sort the individuals according to the objective m
4: **while** i < A **do**
5: $d_{i,m} = \frac{f_{i+1,m} - f_{i-1,m}}{f_m^{max} - f_m^{min}}$
6: $CD_i = CD_i + d_{i,m}$
7: end **while**
8: $cd_0 = cd_A = \infty$
9: end **while**

The parents are then chosen based on their rank and crowding distance values in a binary tournament. An individual is selected if his rank is lower than the others. If they are of the same rank, the individual will be selected if its CD_i is higher than the others. With this, the selected population generates children through crossover and mutation operators. An intermediate population consisting of the current and generated offspring populations is ranked again based on the non-dominance relation explained in the previous subsection (4.2). Individuals with lower ranks have a preference for entry into the next generation of the population. When selecting an individual among two or more within the same level is necessary, the individual with the highest CD_i value will have preference.

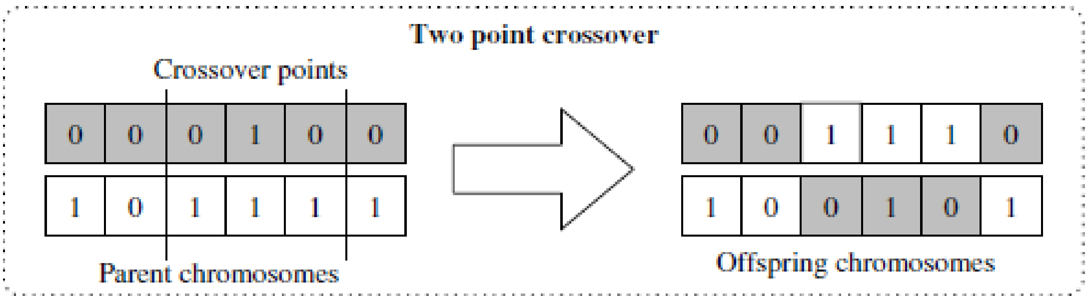
Crossover and Mutation Strategies

Crossover mechanisms are responsible for exploring the search space of the problem in a genetic algorithm. There are several consolidated strategies in recent literature. In this study, was adopted the two-point crossover strategy.

The strategy consists of generating two random position cuts in the parent individuals, which result in three chromosome segments for each parent. The two children are then created by combining the segments mentioned above.

Figura 4

Two-point crossover mechanism



Source: Adapted from Hassanat, Almohammadi, Alkafaween, Abunawas, Hammouri & Prasath (2019).

In addition to recombination operators, which produce offspring by combining parts of two parents, a mutation procedure produces offspring from a single parent. The mechanism consists in, given a probability θ to be executed, a chromosome of the individual is randomly selected, changing its original value. The parameter θ starts at 0% and increases by $\left(\frac{40}{Max_{iteration}}\right)$ in each iteration, reaching a maximum probability of 40% in the last iteration. The mutation procedure is executed after the crossover procedure.

Local Search Scheme

In order to accelerate the convergence of the proposed algorithm, a local search strategy was implemented. Generally, an approach based on a genetic algorithm tends to generate individuals with high similarity as the algorithm iterates. In addition, such heterogeneity reduction can also be verified through the fitness values of the individuals.

To attempt to overcome this obstacle, the local search proposed is executed whenever a given variable called Similarity Coefficient (SC) exceeds a pre-determined parameter of similarity β. The strategy is an adaptation of the mechanism proposed by Huo et al. (2021). The Similarity Coefficient (SC) is calculated as follows:

$$SC_{jk} = \frac{\sum_{i=1}^{A} \varphi(f_{ij}, f_{ik})}{n} \quad (20)$$

where f_{ij} and f_{kj} represent the proportions of investments of each individual i in assets j and k. The calculation of $\varphi(f_{ij}, f_{ik})$ is performed according to Equation 21.

$$\varphi(f_{ij}, f_{ik}) = \begin{Bmatrix} 1, & if\ (f_{ij} = f_{ik}) \\ 0, & otherwise \end{Bmatrix} \quad (21)$$

Thus, the average similarity coefficient SC of the current population can be obtained by Equation 22 as follows.

$$SC = \frac{2 \sum_{j=1}^{P-1} \sum_{k=j+1}^{P} SC_{jk}}{P\ (P-1)} \quad (22)$$

where P is the current population size.

Therefore, when SC exceeds the parameter β, the implemented local search procedure is executed in some individuals of the population, which are randomly selected. In the approach, the local search procedure consists of two steps: first, 15% of the individuals in the population are randomly selected. This percentage was defined through parameterization tests, where values between 5% and 50% were tested, with 5% increments.

Then, a number between one and 20% of the total of assets (chromosomes) is randomly generated, which will indicate the number of moves to be performed for each type of local search operation. This study considered two search structures: SWAP (simple swap between two chromosomes) and INSERTION (insertion of a selected chromosome into another position).

Finally, are executed (randomly) the number of searches moves in the individuals. If the newly generated individual dominates the current individual, the population substitutes this. Otherwise, there is a probability between 0% and 3% that the individual will be replaced anyway. The percentage is initialized with 0% and incremented by $\left(\frac{3}{Max_{iteration}}\right)$ at each iteration of the algorithm.

Data Sets and Performance Metrics

To evaluate the performance of the proposed methods, the instances for PSPs of the OR-Library were used (Chang et al., 2000). The set represents real instances, with data collected from the main financial markets, as the Table 1 shows, and are the most tested instances in the literature, having its optimal frontiers unrestricted detailed and online accessible.

Five instances compose the set, containing 31, 85, 89, 98 and 225 assets. For each asset, is detailed it estimated return and standard deviation, besides a matrix of covariance among all assets.

Table 1

Instances used in tests

Index	Country	Assets
Hang Seng	Hong Kong	31
DAX 100	Germany	85
FTSE 100	UK	89
S&P 100	USA	98
Nikkei 225	Japan	225

Source: The Authors (2022)

In a multiobjective problem, the results can be reported in several ways in the literature. Thus, this section presents a brief explanation of the performance metrics used to compare the results of the methods.

Spacing (S) measures the dispersion of the non-dominated set of solutions obtained compared to the optimal frontier. That is, how much does the distance between each solution of the obtained frontier for the nearest solution belonging to the optimal frontier vary.

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\bar{d} - d_i\right)^2} \tag{23}$$

Where d_i is the Euclidean distance between the point i of the obtained frontier to the nearest point j belonging to the optimal frontier, $\bar{d}$ is the average of all d_i and n, the quantity of present solutions in the obtained frontier. The spacing should be as small as possible so that the set of solutions shows a superior quality; $S = 0$ indicates that all points are equally far from the optimal frontier.

Generational Distance (GD) estimates the distance between the obtained frontier and the optimal Pareto border (Van Veldhuizen & Lamont, 2000). The metric is calculated from the average between the Euclidean distance of the elements of the non-dominated solutions set to the respective nearest points belonging to the optimal Pareto border.

$$GD = \frac{\sqrt{\sum_{i=1}^{n} d_i^2}}{n} \tag{24}$$

Thereby, the lower the value of GD, closer is the set obtained from the optimal frontier, so that, when $GD = 0$, it means that all the set solutions are present in the optimal frontier.

Diversity Metric (Δ) measures the extent of dispersion of the set, that is, how uniformly the points are distributed between the approximation of the set in the objective space. This metric, which is related to the Euclidean distance between the solutions, does not require an optimal frontier (Deb et al., 2002).

$$\Delta = \frac{d_f + d_l + \sum_{i=1}^{n-1}\left|d_i - \bar{d}\right|}{d_f + d_l + (n-1)\bar{d}} \tag{25}$$

Where d_i is the Euclidean distance between consecutive solutions in the obtained frontier, $\bar{d}$ is the average of these distances, while d_f and d_l are the Euclidean distances between the extreme solutions of the obtained frontier with the nearest points of the optimal frontier.

The lower the value of Δ, better the diversification of the set of non-dominated solutions. Then, $\Delta = 0$ indicates that the set is the more evenly distributed as possible.

Besides these, were also considered in the comparison of the results the mean return error (MRE), variance of return error (VRE) and mean percentage error (MPE), mathematically detailed in Chang et al. (2000) and Fernandéz and Goméz (2007).

Computational Results

The results reported in this Section considered the parameters $K = 10$, $\epsilon = 0.01$ and $\delta = 1.00$, similarly to the other methods compared. The performances were reported considering various metrics to understand HNSGA performance better. Thus, the execution of many comparisons was necessary to compare with the main results present in the literature and to report the performance achieved by the proposed method. Table 2 presents the methods developed by each work and the performance metrics used.

Table 2

Methods and Metrics

Paper	Methods	Metrics
Chang et al. (2000)	GA; TS; AS	MPE; Time
Cura (2009)	PSO	VRE; MPE; Time
Sadigh et al. (2012)	PHNN	VRE; MRE
Lwin et al. (2013)	PBIL; PBILDE	MPE; Time
Mishra et al. (2014)	PESA-II; SPEA2; NSGA-II; MOPSO	VRE; MRE; S; GD; Delta
Baykasoglu et al. (2015)	GRASP	VRE; MRE
Bacanin et al. (2014)	MFA	VRE; MPE; Time
Silva et al. (2019)	ANSMOPSO	VRE; MRE; S; GD; Delta; Time
Strumberg et al. (2018)	GI-ABC	VRE; MPE; Time
Kartal (2020)	ABC	VRE; MPE; Time
Kalayci et al. (2020)	HGA-ABC	VRE; MRE

Source: The Authors (2022).

Results for Metrics VRE and MRE

Most works in CCPSP report their results through the VRE and MRE metrics, as reported in Table 2. The first comparative study considered the mentioned metrics, analyzing the performance achieved by the proposed method with the several others proposed in the literature, with the results exposed in Table 3. Concerning the MRE, it was found quite balanced in the compared results. In more minor instances, a tendency towards better results by the methods based on MFA and ABC was evidenced. In larger instances, better performance was observed based on PSO and GA.

The ANSMOPSO and GRASP methods achieved the best results in the VRE metric. However, the proposed method obtained high-quality results, ranking second or third in all instances. Furthermore, the excellent performance achieved by HNSGA for the computational execution time stands out. As the results revealed, the hybridization strategy adopted enabled a higher convergence speed in the search for efficient solutions.

Table 3

Results for Metrics VRE and MRE

Instance	Metric	GA	PESA	NSGA	GRASP	MFA	ANS-MOPSO	GI-ABC	HGA-ABC	ABC	HNSGA
Hang Seng	VRE	1.644	1.523	1.326	1.640	1.238	1.151	1.229	1.639	2.704	1.335
	MRE	0.607	0.762	0.647	0.606	0.471	0.574	0.470	0.6085	0.888	0.614
	Time(s)	18	685	675	27	20	7	19	-	28	22
DAX 100	VRE	7.218	9.282	7.121	6.759	7.256	6.293	7.198	6.781	7.212	7.052
	MRE	1.279	2.221	1.263	1.277	1.379	1.098	1.288	1.278	1.716	1.158
	Time(s)	99	1.606	1.586	86	71	55	65	-	141	89
FTSE 100	VRE	2.866	5.238	2.987	2.430	2.708	2.184	2.635	2.435	4.763	2.809
	MRE	0.328	0.402	0.333	0.324	0.312	0.307	0.310	0.319	0.545	0.328
	Time(s)	106	1.621	1.601	92	94	67	82	-	148	94
S&P 100	VRE	3.480	7.012	3.763	2.521	3.602	2.406	3.599	2.525	5.182	3.280
	MRE	1.226	2.423	0.732	0.906	0.899	0.771	0.881	0.704	1.643	0.852
	Time(s)	126	1.641	1.617	96	148	73	135	-	168	115
Nikkei 225	VRE	1.206	3.098	1.123	0.836	1.201	0.901	1.201	0.819	3.035	0.984
	MRE	5.327	1.231	0.432	0.418	0.489	0.322	0.471	0.423	0.970	0.471
	Time(s)	742	4820	4760	409	367	589	345	-	727	429

Source: The Authors (2022).

Analyzing specifically with GA (Chang et al., 2000), NSGA (Mishra, Panda & Majhi, 2014), and HGA-ABC (Kalayci, Polat & Akbay, 2020) methods, which consider genetic algorithm structures in their approaches, the proposed approach showed superior performances in most metrics and instances. Regarding the MRE and VRE metrics, HNSGA achieved superior results in all instances compared to GA and NSGA, including relative computational time. These results demonstrate that the hybridization proposed in the method and the ranking strategy adopted allowed for obtaining better quality solutions at a lower computational cost, accelerating the convergence capacity in the search process.

Regarding HGA-ABC, HNSGA achieved superior performance in instances of larger dimensions, while HGA-ABC obtained better results in instances with smaller assets. This fact demonstrates that the proposed method has a higher search space exploration capability, allowing it to achieve better quality solutions on high dimensional instances. Such capacity is maximized both by the local search strategy adopted and by the parameters stipulated for the perturbation mechanisms. It represents a substantial contribution of the study to the real world, since there is an increasing entry of organizations in the financial market, increasing the instances considerably in real problems.

Thus, the results corroborate that hybrid heuristic strategies have achieved superior results for multi-objective PSP problems. Such fact is demonstrated by, in addition to HNSGA and HGA-ABC, the other two methods proposed in the literature that achieved competitive results were ANSMOPSO (Silva et al., 2019) and GI-ABC (Strumberg et al., 2018), both also hybrid heuristic approaches.

Results for Metrics MPE and MedPE

In comparing the results obtained for the problem, was also considered the MPE metric. The performance of HNSGA was compared with the approaches proposed by Chang et al. (2000), Xu et al. (2010), Lwin and Qu (2013) and Silva et al. (2019). Table 4 shows that the proposed method produced competitive results in all instances tested.

Table 4

Results for Metrics MPE and MedPE

Instance	Assets	Metric	GA	PBIL	PBILDE	ANS-MOPSO	HNSGA
Hang Seng	31	MPE	1.0974	1.1026	1.1431	1.0520	1.0922
		MedPE	1.2181	1.2190	1.2390	0.7917	1.1225
DAX 100	85	MPE	2.5424	2.5163	2.4251	2.1570	2.3235
		MedPE	2.5466	2.5739	2.5866	2.0184	2.2951
FTSE 100	89	MPE	1.1076	0.9960	0.9706	0.9128	0.9355
		MedPE	1.0841	1.0841	1.0840	0.6642	0.9291
S&P 100	98	MPE	1.9328	2.2320	1.6386	1.6176	1.6135
		MedPE	1.2244	1.1536	1.1692	1.2170	1.1955
Nikkei	225	MPE	0.7961	1.0017	0.5972	0.6178	0.6035
		MedPE	0.6133	0.5854	0.5896	0.2273	0.5312
Avg.	-	MPE	1.4953	1.5697	1.3549	1.2914	1.3136
		MedPE	1.3373	1.3232	1.3337	1.0437	1.2147

Source: The Authors (2022).

Results for Metrics S, GD and Delta

Finally, the performance of the proposed approach was also analyzed using the S, GD, and Δ metrics. The comparison was performed strictly on the Nikkei 225 instance because the other authors reported their results on this instance, as shown in Table 5.

Table 5

Results for Metrics S, GD and Δ - Instance Nikkei 225

Metric	PESA-II	SPEA2	NSGA-II	MOPSO	ANSMOPSO	HNSGA
S	2.33E-5	6.40E-6	4.70E-6	3.48E-6	3.15E-6	4.12E-6
GD	1.76E-2	1.02E-3	6.72E-3	1.45E-4	1.31E-4	1.74E-3
Δ	0.593	0.386	0.296	0.133	0.312	0.298

Source: The Authors (2022)

The results obtained by HNSGA presented efficient performances for most of the metrics considered, reaching quality results similar to the best approaches. Furthermore, the method achieved the best results for the S metric, responsible for measuring the dispersion of the non-dominated set of solutions obtained compared to the optimal frontier.

In a specific comparison with the NSGA-II method, the proposed approach is superior for the main performance metrics considered in this work, especially GD.

The superior results in GD indicate that adopting the local search mechanisms in the proposed hybridized approach increased the convergence capability of the method relative to the traditional NSGA-II proposed by Mishra, Panda and Majhi (2014). In turn, the results obtained for S suggest that the extended mechanisms in the proposed method have increased the exploration capability of the solution search space, given that, relative to NSGA-II, there was a significant improvement in the approximation of the obtained efficient boundaries to the optimal boundaries of the instances.

Finally, the performance achieved in the Δ metric suggests no significant change in the result, demonstrating that both methods presented an ability to achieve similar amplitudes in their efficient frontiers. It indicates that the strategies adopted in the method do not influence the increase or decrease in the amplitude of the frontiers obtained.

The method demonstrated efficiency in solving the CCPSP, having achieved results competitive with the main approaches reported in the literature. The developed heuristic demonstrated fast convergence to high-quality solutions from a computational perspective, as evidenced by the reported execution times, especially in large instances.

Concluding Remarks

An extensive literature review was conducted in this work, aiming to identify the main approaches developed to address CCPSP. Additionally, the review allowed consolidation of the main performance metrics considered in the literature.

Thus, the present study proposed developing a hybrid evolutionary heuristic approach to deal with CCPSP, which is based on a multi-objective genetic algorithm that uses local search structures. The approach was called Adaptive Hybrid Non-dominated Sorting Genetic Algorithm (HNSGA).

The approach was tested using PSP instances from OR-Library, considering several metrics reported in the literature for evaluating its performance. The results showed that HNSGA achieved competitive or superior results compared to the main works reported in the literature in all metrics considered in the study.

The proposed heuristic provided fast computational convergence to high-quality solutions, as seen by the observed execution times, particularly in large instances. Thus, the approach demonstrated high performance in solving the CCPSP.

This work allowed us to achieve significant scientific advances by addressing optimization methods applied to financial markets, a widely discussed and studied topic nowadays. In addition, the study also allowed for advances in the approach developed, which allowed for high-quality results in a feasible computational time.

As future research, the study of optimization approaches applied to CCPSP that consider transaction costs, new risk measurements, and other market operational constraints will possibly have significant importance in several real applications.

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