A 3D study of the contact surface developed by the contact between the tires and the structural pavements

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The basic aim in this work is to present a new technique to analyze the contact surfaces developed by the contact between the tires and the structural pavements by numerical simulations, using 3D finite element formulations with contact mechanics. For this purpose, the Augmented Lagrangian method is used. This study is performed just putting the tires on the structural pavement. These tires and the structural pavement are discretized by finite elements under large 3D elastoplastic deformation. The real loads (of aircrafts, trucks or cars) are applied directly on each tire and by contact mechanics procedures, the real contact area between the tires and the pavement surface is computed. The penetration conditions and the contact interfaces are investigated in details. Furthermore, the pressure developed at the contact surfaces is automatically calculated and transferred to the structural pavement by contact mechanics techniques. The purpose of this work research is to show that the contact area is not circular and the finite element techniques can calculate automatically the real contact area, the real geometry and its stresses and strains. In the end of this work, numerical results in terms of geometry, stress and strain are presented and compared to show the ability of the algorithm. These numerical results are also compared with the numerical results obtained by the commercial program ANSYS.

Key words: Airport pavement. Contact mechanics. Contact surface. Elastoplasticity. Finite elements.

1 Introduction

Several formulations concerning treatment of the contact interface have been presented in the literature, especially when the contact interface of two contacting bodies is rough. Two main lines can be followed to impose contact conditions in normal direction. One is regarding to the formulation of the non-penetration condition as a purely geometrical constraint, and other, to the development of constitutive laws for the micromechanical approach within the contact area, which yields a response function for the normal stresses in terms of parameters, such as the mean real area of contact or the current mean plane distance (BANDEIRA; WRIGGERS; PIMENTA, 2001; 2004; BANDEIRA; PIMENTA; WRIGGERS, 2003; WRIGGERS, 1995). For the tangential stresses, the same situation arises for sticking contact interface where either a geometrical constraint equation or a constitutive law for the tangential micro displacement can be formulated. For sliding between the bodies in contact, a constitutive equation has to be formulated in form of an evolution equation.

The basic aim in this work is to present a new technique and procedures to model pavement structures, using contact mechanics techniques to define the contact area and its forces transferred to the pavement. Secondarily, it is going to be studied the results in terms of stresses and strains. In this simulation, it was used the parameters of the aircraft EMB 145 (regional jet) (BANDEIRA; MERIGHI; FORTES, 2006; BANDEIRA; MERIGHI, 2005).

For this purpose, taking into account elastoplastic constitutive equations, a finite element approach using contact mechanics formulations are applied to simulate pavement structures under finite three-dimensional deformation.

The Augmented Lagrangian method is used to solve the 3D frictional contact problems between the tires and the top pavement structure surfaces. At the contact surface, high-pressures occur, which cannot be treated adequately by standard penalty procedures (BERTSEKAS, 1995). The mathematical theories concerning Augmented Lagrangian method in the context of mathematical programming problems, subjected to equality and inequality constraints are well established by Bertsekas (1995).

The algorithms for frictional contact, developed between the tire and the asphalt, are derived basing on a slip rule, using backward Euler integration, like in plasticity. The complete frictional contact mechanics formulation in three-dimension is presented in Bandeira, Wriggers and Pimenta (2004), and in Bandeira, Pimenta and Wriggers (2003). The cases of node-to-surface, node-to-edge and node-to-node contact are consistently considered in order to describe the contact between discretized surfaces in a correct manner.

The finite element program, CMAP (Contact Mechanics Analysis Program), is based on a C++ code developed by Bandeira, Wriggers and Pimenta (2004). All formulations presented here yield asymptotic quadratic rates of convergence within a Newton equation-solving strategy, owing to the exact linearization employed. Furthermore, the algorithm is stable, has a short evaluation time and thus high performance. All numerical examples given are based on three-dimensional calculations.

To solve the unconstrained optimization, the Newton method is used to resolve the nonlinear system, and a sophisticated Preconditioned Bi-Conjugate Gradient Method (PBCG) for sparse matrices is used to solve the linear system.

A three-dimensional brick element with eight nodes is used for the treatment of finite elastoplastic deformation of the contacting surfaces. This finite element formulation will not be presented in this article, because it is thoroughly described in the literature.

The procedure to calculate stresses, deformations and displacements are performed by placing the tires on the top pavement surface and by applying the contact mechanics procedures to get the numerical results. These procedures are presented in details in this article. In the end of this work, numerical simulations are presented to show the ability of this algorithmic. Some classical numerical examples are also compared with the theoretical one and with the elastic constitutive law.

2 Contact mechanics techniques to solve pavement structures

It is important to mention here, that the formulations of the finite element method for the continuum, in special the brick element, are fully present in the literature.

The contact mechanics theory, like the principle of virtual work, contact kinematics, contact contribution to the weak form and the complete formulation of frictional contact mechanics, are completely developed in details in Bandeira, Wriggers and Pimenta (2004) and in Wriggers (1995).

Also, the formulation and algorithm of the constitutive equation for elastoplasticity developed by von Mises in the principal axis are presented in Bandeira, Pimenta and Wriggers (2003).

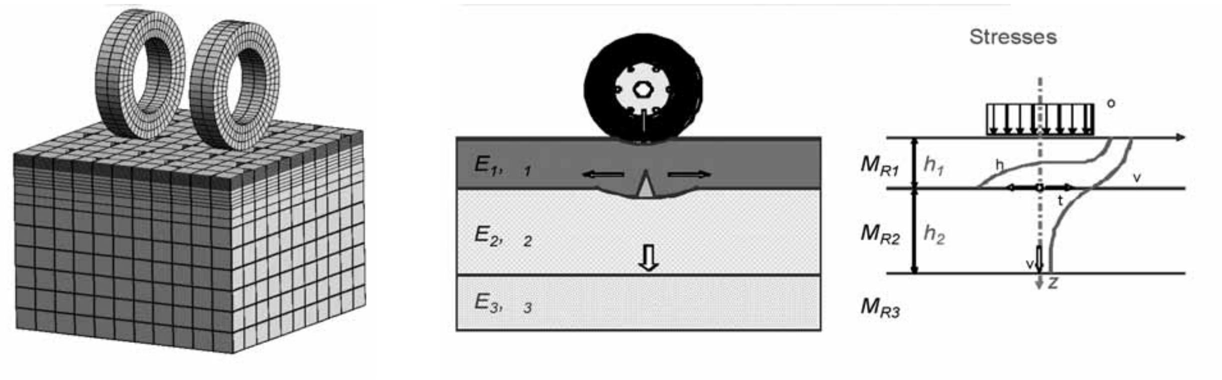
In this section, the model to simulated pavement structures is presented.

2.1 Tridimensional model to simulated pavement structures

The basic aim in this paper is to solve pavement structures problems using contact mechanics techniques with the finite element method. For this purpose, the pavement structure has to be modeled by the discretization of the several layers of the pavement structure (Figure 1b). At the asphalt concrete surface, two aeronautical tires classified as H41x 16 – 20 22R, each one with 406 mm width and 104 cm height, are placed 87 cm between them; inflation pressure equal 154 psi or 1,083 MPa (Figure 1c). This problem summarizes the model that consists of three deformable bodies in contact (Figure 1a).

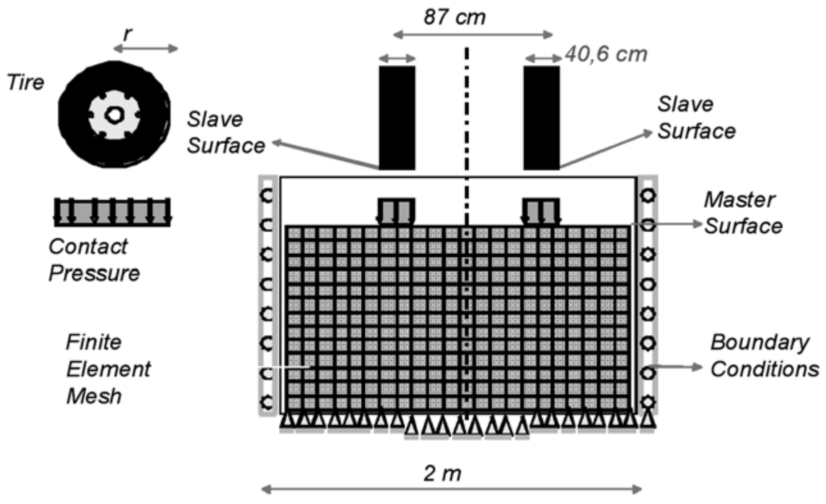
The pavements and the tires are discretized using the brick elements with eight nodes (Figure 1a). That is the reason why our contact interfaces are discretized by contact elements with four nodes. Each soil layer has its material parameters defined by the elasticity modulus (E), the Poisson ration (ν) and the initial yield stress (σ_{yo}). Here it is assumed that the pavements should be modeled by a two meters solid, in each horizontal side, and the real thickness of the pavement structure in the vertical direction (Figure 1b). Regarding the boundary conditions of the pavement, the last surface layer has its displacements completely restrained. The sides of the pavement model have no restraints on the vertical direction, but they are completely restrained on the other two possible displacements (Figure 1c).

The procedure is performed just putting the tires on the asphalt surface. In the contact discretization, the asphalt surface is defined as the master surface, and the tire surfaces in contact with the asphalt are defined as slave surfaces. Then, the simulation is performed and, in the equilibrium configuration, the results lead to the correctly phenomenon. In the end of the numerical simulation, the results are compared in terms of stress with the classical pavement formulations and with the results obtained by the classical software.



(a) 3D model

(b) Pavement cross section (yz) and theoretical stress



(c) Pavement cross section (xz)

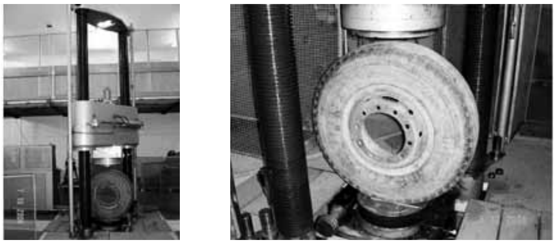
Figure 1: Motivation of the real pavement model
Source: The authors.

In Figure 2, is presented the configuration of the main gear.

2.2 Experiment to obtain the tire parameters

Material parameters of the tire are obtained by experimental procedures as shown in Figure 3. The results obtained are plotted in Figure 4. After some analysis, the elasticity modulus lays around 400 MPa, and the Poisson ratio around 0,35. It is important to mention that these parameters are used in the numerical examples.

Figure 2: Configuration of the main gear
Source: Embraer. Available in: <http://mediamanager.embraer.com.br/portugues/content/busca/detalhe_publico.asp?que_pagina=3&pagina_anterior=categorias>.



Test equipment

Truck tire

Figure 3: Experimental procedure to obtain the material parameters of the tire

Source: The authors

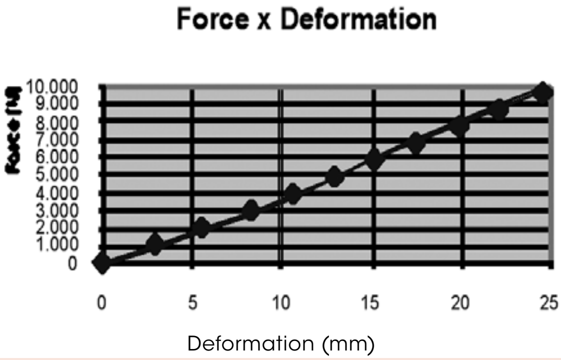


Figure 4: Constitutive equation of the tire

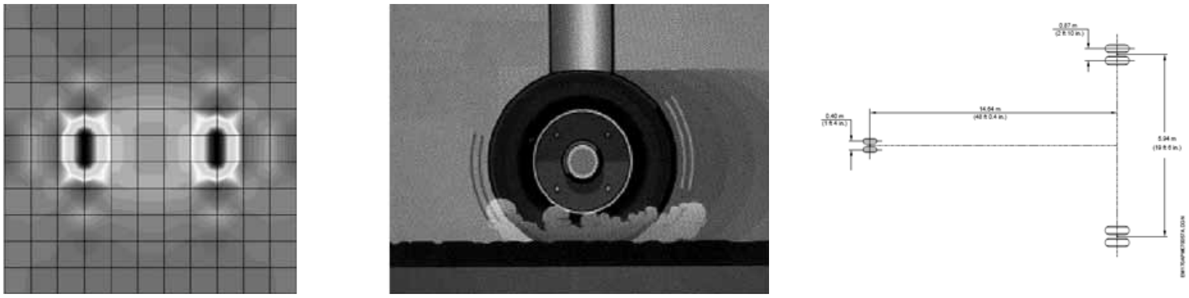
Source: The authors.

2.3 Contact area and contact pressure

In standard pavements formulations, it is important to give the contact area defined by the contact between the tires and the pavement, demarked, for example, by a circle with 10,8 cm ray and a pressure around 0,56 MPa (81 psi). So analyzing each tire, the resultant load developed at the contact interface is around 20.520 N, i.e., [0,56 MPa × π × (108 mm)²].

In this work, the loads are applied directly on the EMB-195 tires (154 psi – 1,08 MP), each one assuming proximally 124.569 N, i.e., [524.500 N/airplane × 0,95 ÷ 4 tires] with contact area defined as a circle with 191,61 mm ray, i.e., [124.569 N ÷ 1,083 MPa = $\pi \times r^2$]. The algorithm of contact problem defines by itself the contact interface (Figure 5a). The loads are applied directly on the tires and, furthermore, are transferred to the pavement by the contact problem. It is done using the finite element formulation with a four-node-contact-element. The contact formulations of node-to-surface, node-to-edge and node-to-node are already reported in the literature, presented in details in Bandeira, Wriggers and Pimenta (2004), and will be not presented in this article.

In Figure 5a, it can be seen the real contact area developed between the two tires and the pavement. It is calculated automatically and the geometric form is elliptical. In that picture, the higher stresses are developed exactly at the contact interface between the tires and the pavement. In Figure 5c, the tires geometry of the aircraft are presented.



(a) stress σ_{13}

(b) stress σ_{13}

(c) tires geometry

Figure 5: Contact area

Source: The authors.

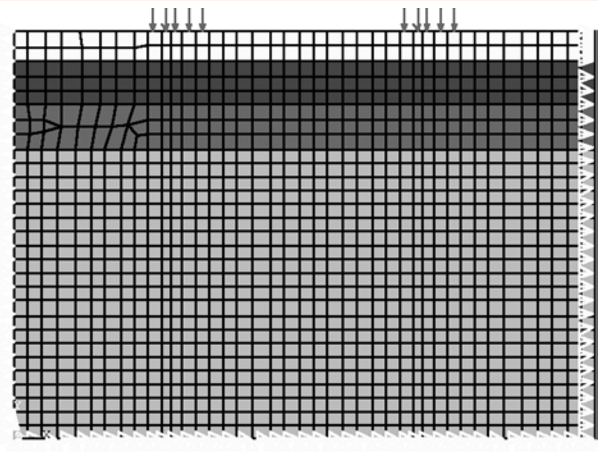


Figure 6: Contact area
Source: The authors

2.4 Two dimensional structural solid using ANSYS

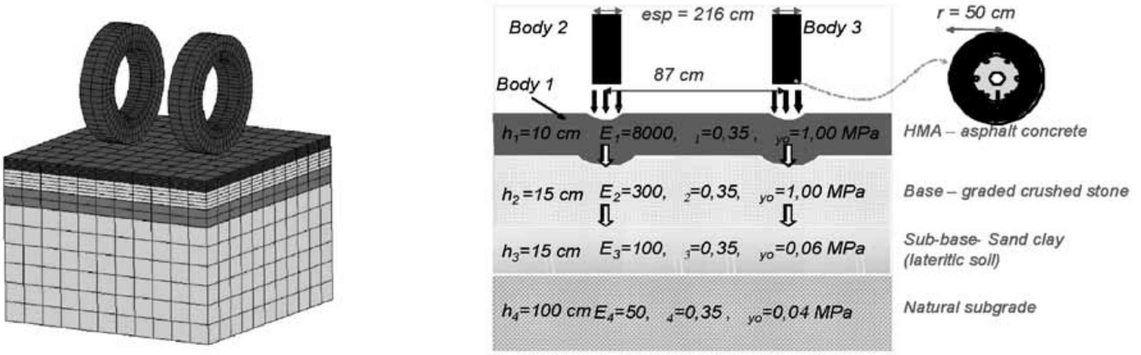
To compare the numerical results obtained by the 3D model, described in section 1.1, the ANSYS program is used to model a 2-D structural pavement by the PLANE42 element. This element can be used either as a plane element (plane stress or plane strain) or as an axisymmetric element. Here, the plane stress model is used. The element is defined by four nodes having two degrees of freedom at each node: translations in the nodal x and y directions. The element has plasticity, creep, swelling, stress stiffening, large deflection, and large strain capabilities. The 2D model is illustrated in Figure 6.

The boundary conditions shown in Figure 6 are the same presented in section 1.1. The pressure is applied on the top surface. It is important to mention that in this model it doesn't contain contact, i.e., the loads are applied directly on the pavement surface, as shown in Figure 6. The standard finite element simulation is used in this 2D model.

3 Numerical simulations

The case analyzed in this research concerns of a kind of structural pavement, normally used in São Paulo state regional Airport (FORTES; MERIGHI; BANDEIRA, 2006; UDDIN; GARZA, 2002). See Figure 7a for a representative cross section of this structural pavement. In this figure, the material parameters and the geometry dimensions of each soil layer are defined in detail.

In this section, numerical examples are present to show the ability of the contact mechanics techniques to solve pavement structures. In the Figure 6a, it is illustrated the real 3D model, that consists in a contact problem between two tires and a pavement, as explained in this article.



(a) Pavement with four layers

(b) Different materials used

Figure 7: Numerical simulation of flexible structural pavement
Source: The authors

The pavements are modeled by a cubic solid that represents the pavement portion, demarked by two meters, in the horizontal side, and soil layers in the vertical direction. The finite element mesh used to solve this problem is illustrated in Figure 7a. This numerical problem consists of 48.888 degrees of freedom, 13.885 of brick elements, 64 slave surfaces and 529 master surfaces. The penalty parameters used in this friction contact problem are regarded with 1.0E+9 in normal direction and 1.0E+3 in tangential directions. It is important to mention that, this finite element problem has quadratic rate of convergence, a short evaluation time and high performance.

The numerical simulation presented is a pavement structure model, as shown in Figure 7b. It is considered the first pavement layer, defined by an asphalt with thickness of 10 cm and material parameters defined by E = 8.000 MPa, $\nu = 0{,}35$ and $\sigma_{yo} = 1{,}00$ MPa. The second one is the *base – graded crushed stone* with thickness of 15 cm and material parameters defined by E = 300 MPa, $\nu = 0{,}35$ and $\sigma_{yo} = 1{,}00$ MPa. The third one is the *sub-base - Sand clay (lateritic soil)* with thickness of 15 cm and material parameters defined by E = 100 MPa, $\nu = 0{,}35$ and $\sigma_{yo} = 0{,}06$ MPa. And the last one is a *natural subgrade* with thickness of 100 cm and material parameters defined by E = 50 MPa, $\nu = 0{,}35$ and $\sigma_{yo} = 0{,}04$ MPa.

To solve this problem, the contact mechanics techniques are applied. To show the numerical results, the stresses and displacements are plotted in a special cross section as shown in Figure 8a. In this cross section, it is possible to understand the physical influence of the two tires on the structural pavement and see the behavior of displacements and stresses. The vertical displacement is shown in Figure 8b. The stresses σ_{11}, σ_{22}, σ_{33}, σ_{12}, σ_{23} and σ_{13} are shown in Figure 8c to 8h, respectively.

The same numerical results are presented in other cross section, as shown in Figure 9a. The vertical displacement is shown in Figure 9b. The stresses σ_{11}, σ_{22}, σ_{33}, σ_{12}, σ_{23} and σ_{13} are shown in Figure 9c to 9h, respectively.

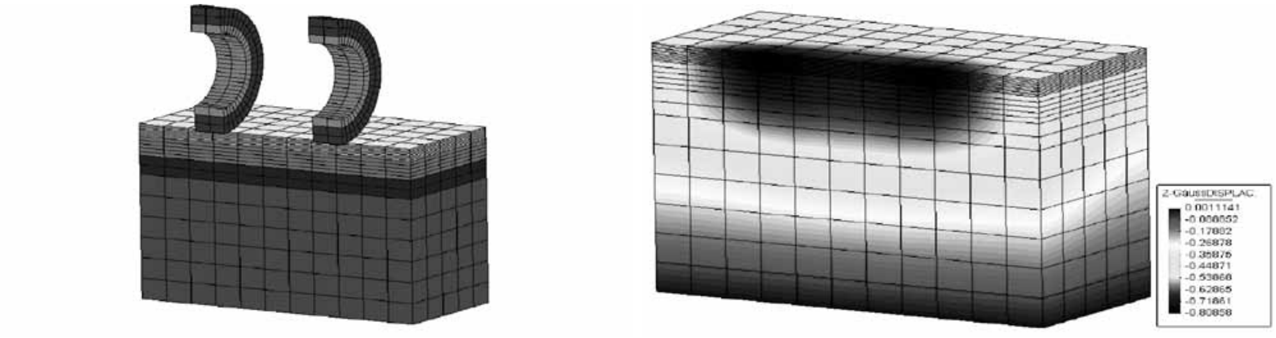
It is important to mention that the soil under the asphalt has a small resistance, as shown in its material parameters. It is the reason that the stresses are not continuous at the interface between the second soil layer and the asphalt. The shear stress σ_{12} goes to zero.

The 2D model presented in section 1.4 is simulated by the ANSYS program. The numerical results are plotted in Figure 11. See Figure 11a for the normal displacement and Figure 11b for the σ_{33} stress.

It is important to compare the numerical results obtained by the Contact Mechanics Analysis Program (CMAP) developed by Bandeira, Wriggers and Pimenta (2004), with the 2D Plane Stress ANSYS solution. Analyzing the normal displacement, in Figures 8b and Figure 11a, we can see that the behavior of the displacement is almost the same, but the 3D model using contact mechanics gets better solution because, in this numerical simulation, does not exist simplifications, i.e., the pressures developed at the pavement surface distributed automatically and correctly, the pressure at the structural pavement. The same occurs in the σ_{33} stresses (Figure 8e and Figure 11b). Again, with the 3D model with contact mechanics we can get results with good agreements with the real phenomena.

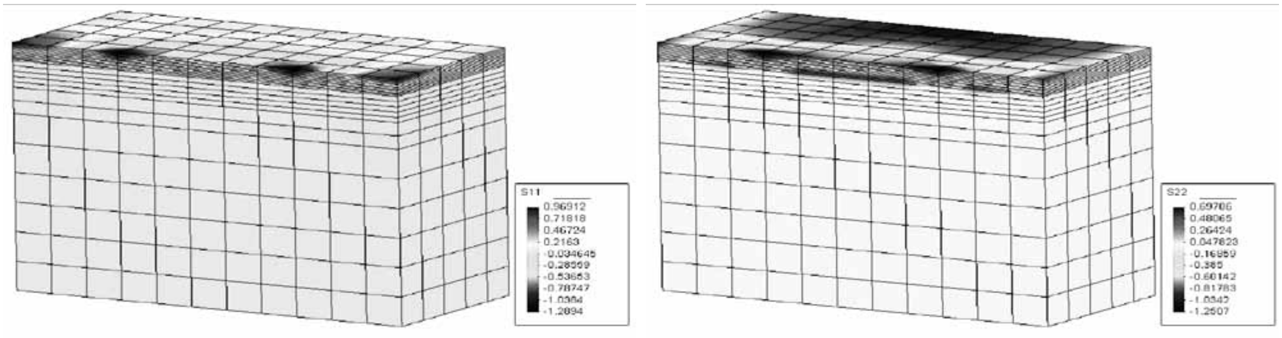
4 Conclusions

The basic aim of this paper is to present a model to solve pavement structures using contact mechanics techniques under large 3D deformation. This is done numerically. The model consid-



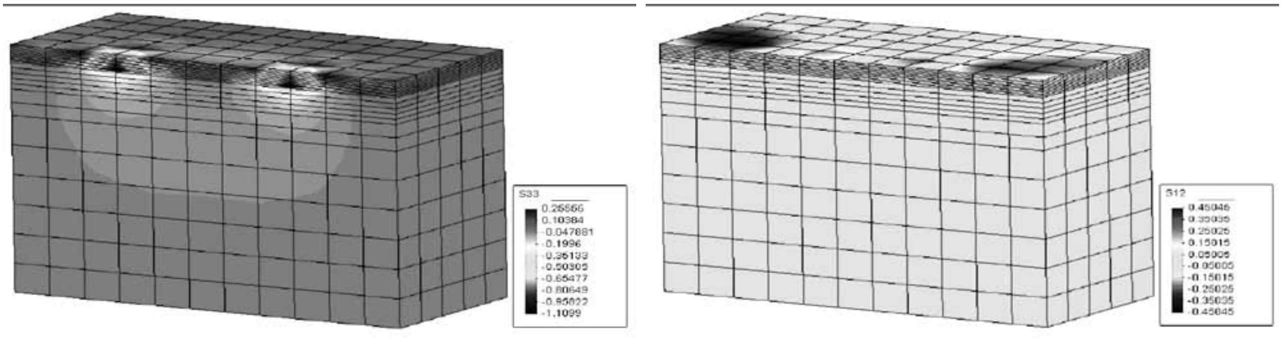
(a) Pavement with four layers

(b) Displacement in direction Z



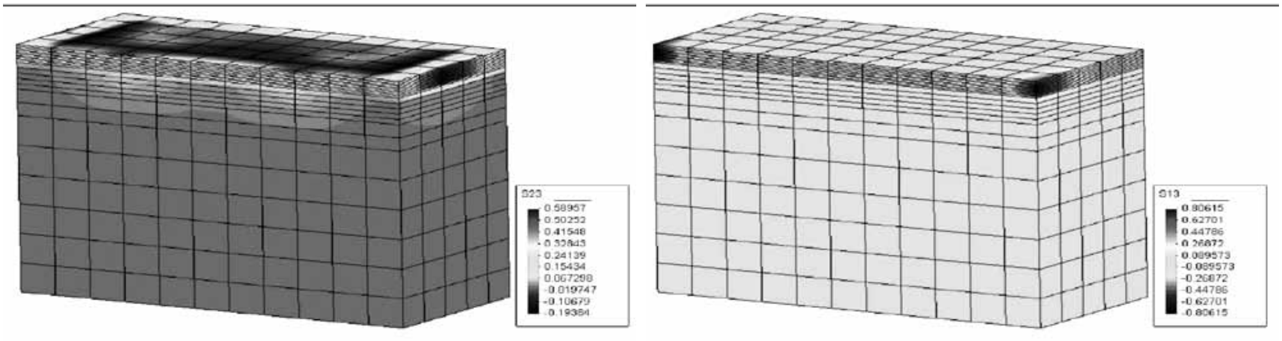
(c) Strees $\sigma11$

(d) Strees $\sigma22$



(e) Strees $\sigma33$

(f) Strees $\sigma12$

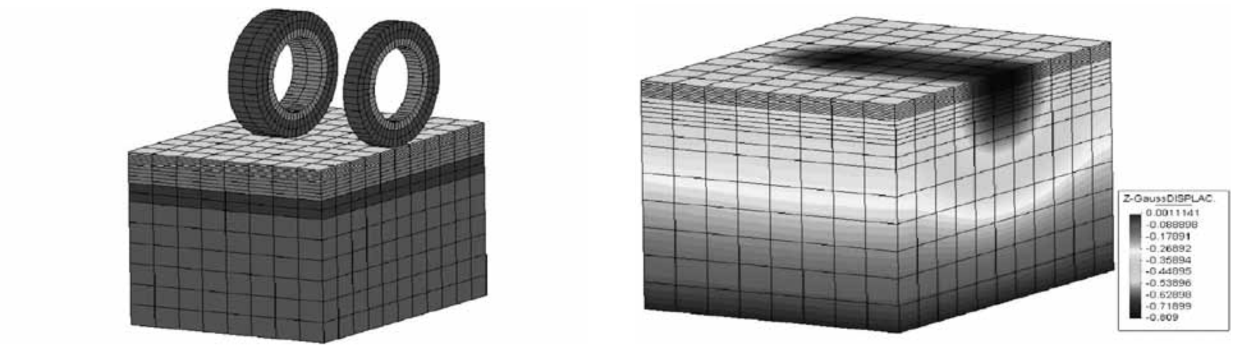


(g) Strees $\sigma23$

(h) Strees $\sigma13$

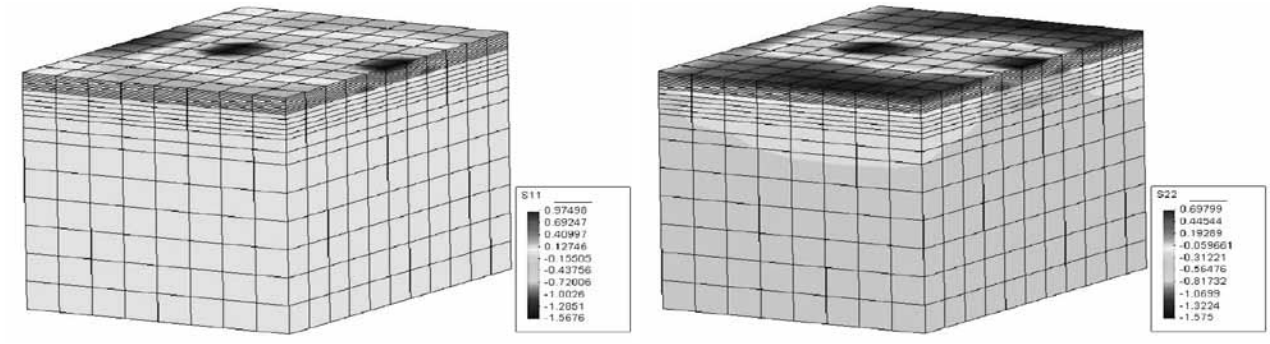
Figure 8: Numerical simulation of flexible structural pavement

Source: The authors



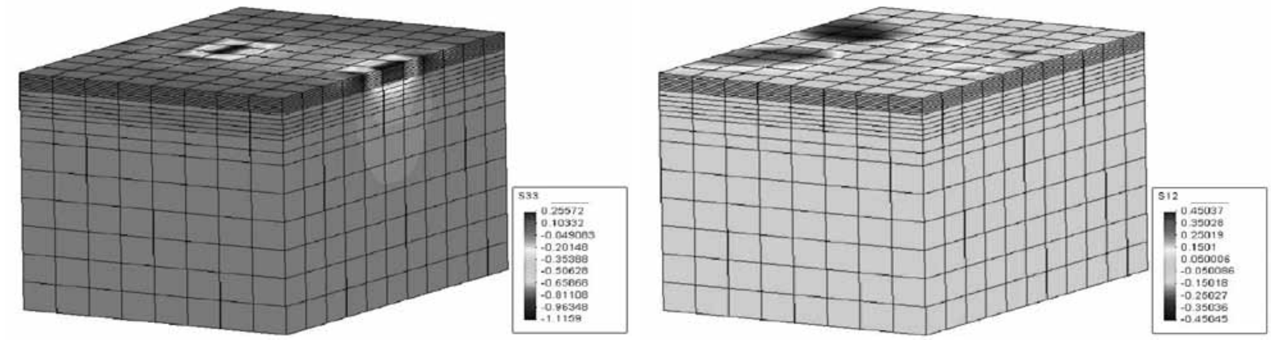
(a) Pavement with four layers (cross section)

(b) Displacement in direction Z



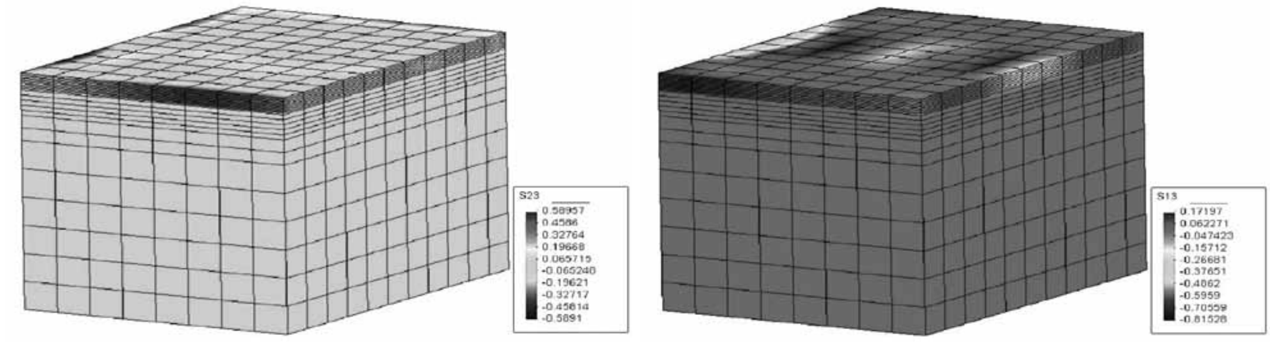
(c) Strees $\sigma 11$

(d) Strees $\sigma 22$



(e) Strees $\sigma 33$

(f) Strees $\sigma 12$



(g) Strees $\sigma 23$

(h) Strees $\sigma 13$

Figure 9: Numerical simulation of flexible structural pavement

Source: The authors

ered is based on the assumption of a Von Mises elastoplastic material. This formulation was derived based on exact linearization, considering the possible sliding of slave node from one surface to the adjacent one.

It is important to mention that the numerical results differ from the theoretical one. The numerical results obtained by the contact mechanics simulation is better than the other one obtained by a simple model (Plane Stress), because our contact area and the pressures developed between the tires and the surface of the pavement structure is obtained by the contact formulation. The authors can conclude that the results obtained from a sophisticated 3D finite element models with contact mechanics have more accuracy regarding other simplification models.

Further investigations will include a study of the strain hardening coefficient of the materials presented in the plasticity theory. Other suggestion has developed a new constitutive equation that represents the real phenomenon at the contact interface between the tire and the asphalt surface, like in micromechanics contact formulations.

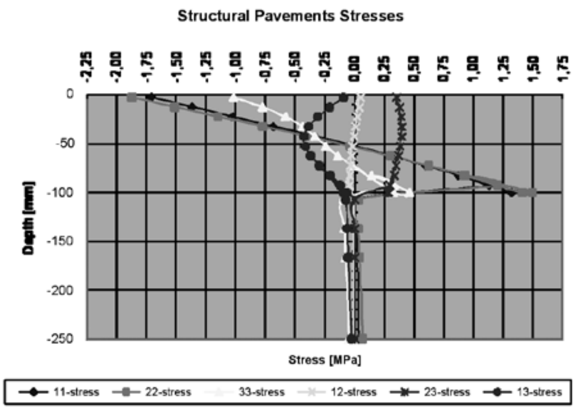
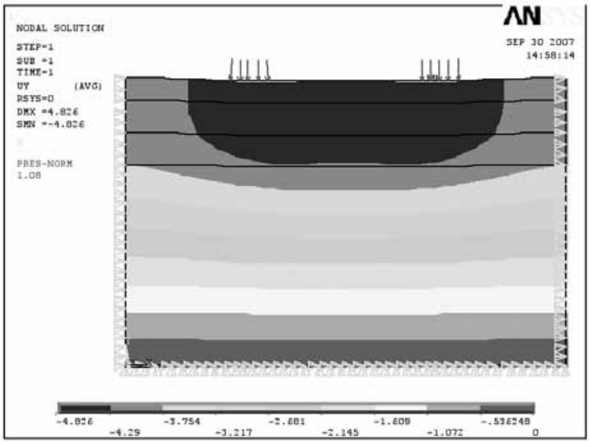
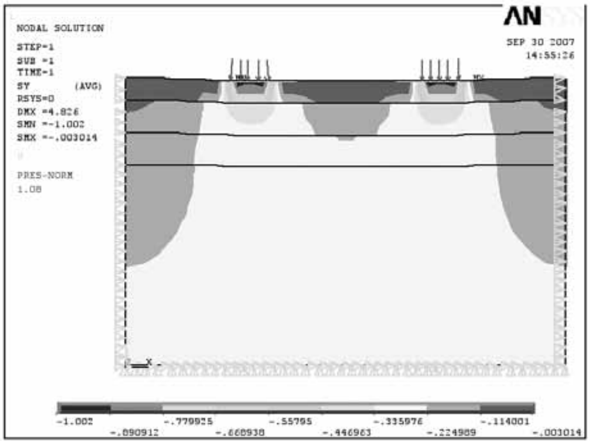


Figure 10: Pavement structures stresses for a point under the tire
Source: The authors



(a) Displacement



(b) Stress $\sigma_y(\sigma_{33})$

Figure 11: 2D model – Plane Stress Formulation
Source: The authors

References

BANDEIRA, A. A.; MERIGHI, J. V. Numerical simulation of 3D structural pavements – A finite element approach using contact mechanics techniques. In: *European Airport Pavement, 1.*, Amsterdam, Netherlands: CROW, 2005. v. 1.

BANDEIRA, A. A.; MERIGHI, J. V.; FORTES, R. M. A pavement structures study based in terms of stress and strain using a finite elements procedure under large 3D elastoplastic deformation. In: *International Airports Conference: Planning, Infrastructure & Environment*, 2., São Paulo: ANDIT, 2006.

BANDEIRA, A. A.; PIMENTA, P. M.; WRIGGERS, P. Numerical simulation of 3D contact problems under finite elastic-plastic deformation. In: *Us National Congress on Computational Mechanics, 7.*, Albuquerque, New Mexico: Rensselaer Polytechnic Institute, 2003. v. 7.

BANDEIRA, A. A.; WRIGGERS, P.; PIMENTA, P. M. Homogenization methods leading to interface laws of contact mechanics – A finite element approach for large 3D deformation using Augmented Lagrangian Method. In: *Contact Mechanics International Symposium, 3.*, Peniche: Lisbon University, 2001. v. 3.

______. Numerical derivation of contact mechanics interface laws using a finite element approach for large 3D deformation. *IJNME*, England, Vol. 59, n. 2, p. 173-195, 2004.

BERTSEKAS, D. P. *Nonlinear programming*. Belmont: Athena Scientific, 1995.

EMBRAER. Disponível em: http://mediamanager.embraer.com.br/portugues/content/busca/detalhe_publico.asp?que_pagina=3&pagina_anterior=categorias.

FORTES, R. M.; MERIGHI, J. V.; BANDEIRA, A. A. Laboratory studies on performance of porous concrete. In: *International Symposium on Concrete Roads, 10.*, Brussels. *Proceedings*... Brussels: CEMBUREAU – the European Cement Association, 2006. v. 10.

UDDIN, W.; GARZA, S. In situ material characterization of pavement-subgrade systems using FWD data and validation by 3D-FE-simulations. In: *The 2002 FAA Airport Technology Transfer Conference*, Atlantic City,. New Jersey, p. 1-12, 2002.

WRIGGERS, P. Finite Element Algorithms for Contact Problems. *Archives of Computational Methods in Engineering*, v. 2, n. 4, p. 1-49, 1995.

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