A linear programming optimization model
applied to the decision-making process of a
Brazilian e-commerce company

Um modelo de otimização de programação
decisional aplicado ao processo de tomada de decisão
de uma empresa brasileira de comércio eletrônico

Abstract
The decision-making process is not always simple and requires a
more careful analysis to maximize the company’s revenue. This
paper proposes a linear programming model applied to the
decision-making of the section of quality monitoring and packaging
of a Brazilian company of e-commerce, in which the simplex
method was used to maximize the company’s revenue from
historical time data of the activities for each type of product. From
the results, it was verified which products should be prioritized,
providing a revenue of US$ 74,681.50. In addition, a simulation was
applied to include two employees in the process, which would
provide a 32.76% increase in the company’s profitability and a new
revenue of US$ 99,145.00.

Key-words: Linear programming; Optimization; Simplex method; E-
commerce; Decision-making.

Resumo
O processo de tomada de decisão nem sempre é simples e requer
uma análise mais cuidadosa para maximizar a receita da empresa.
Este trabalho propõe um modelo de programação linear aplicado à
tomada de decisão do setor de monitoramento e embalagem da
qualidade de uma empresa brasileira de e-commerce, no qual o
método simplex foi utilizado para maximizar a receita da empresa a
partir de dados históricos das atividades para cada tipo de produto.
A partir dos resultados, verificou-se quais produtos deveriam ser
priorizados, proporcionando uma receita de US$ 74,681,50. Além
disso, uma simulação foi aplicada para incluir dois funcionários no
processo, o que proporcionaria um aumento de 32,76% na
lucratividade da empresa e uma nova receita de US$ 99,145,00.

Palavras-chave: Programação linear; Otimização; Método Simplex;
E-commerce; Tomada de decisão.
1 Introduction

The scarcity of resources and high competitiveness cause companies to seek improvement of their processes. However, focusing only on the production process cannot bring satisfactory results (Almeida et al., 2018a). Prioritizing other sectors, such as inspection and assembly, favours the outcome of the process by maximizing the company’s efficiency. Therefore, many companies seek to make decisions through process planning in which continuous improvement programs are often applied.

Decision-making is characterized by being critical for organizations (Freitas et al., 1997) and can be applied to situations of risk or uncertainty. According to Motta and Vasconcelos (2002), the rational decision model created from the classical economics is characterized at different stages, such as: problem identification; alternative solutions development; alternatives solutions comparison; decisions implementation. Many papers address the decision-making and administrative process, such as: Sousa et al., (2017); Silva et al., (2015); Triches et al., (2015); Araújo et al., (2009); Machado (1999); Ocanã (1999); Freitas et al., (1997) and Gasson (1973). At the same way, several papers use mathematical methods and applications to optimize results, such as: Almeida et al., (2018b); Perdonà et al., (2017); Sousa and Soares, (2014); Longaray and Damas, (2013), Bandeira et al., (2010).

The decision-making process is often not intuitive, requiring detailed analyses to the best choice. Proposing analytical solutions can bring satisfactory results, since the decision theory has a mathematical foundation. Thus, modelling the problem through linear programming (LP) can bring satisfactory results, since, according to Cooper, Edgett, and Kleinschmidt (2000), quantitative models are widely used by organizations.

The Linear Programming (LP) is a technique used to find an optimal point for several variables from an established function, satisfying a set of constraints. LP stands out as one of the most efficient techniques for management tools, in which it is applied in several sectors as in Hall (2010), Nash (2000), Yang and Lin (2000). Among the algorithms used in linear programming, the Simplex algorithm is the most used for LP problem solving.

It is possible to find in the literature several studies involving mathematical modeling applied for decision making, such as Meng et al. (2014), who developed a study in the area of B2B e-business, proposing a stochastic programming model in which the objective function is to minimize the total cost. This work is similar to the study proposed by our paper. Meng et al. (2014) used the linear decision rule and analyzed the central allocation among all retailers in the supply chain through the dual theorem and cooperative game theory. It is equally important to cite the work of Fathian, Sadjadi and Sajadi (2009). These authors used another programming technique: the model based on Geometric Programming (GP) to analyze the price and quality of service for e-business companies. In this work, GP was applied to determine the optimum solution of the model that considers the price and the marketing practiced in Internet-based service providers.

Therefore, applying mathematical techniques (such as LP) to aid the decision-making process is widely studied, especially in a promising context, such as e-commerce. Using already-established techniques (such as the Simplex
method) in a business-to-customer (B2C) model can provide interesting contributions to the e-business area, which is on the rise on the world stage. This paper seeks to use a linear programming approach in the decision-making process applied to a sector of a particular Brazilian e-commerce company. Among the various sectors, the study refers to the sector of quality monitoring and packaging for specific products. In this way, the Simplex optimization method was used to maximize the company’s revenue from historical time data of the activities for each type of product. The data collection was performed during a determined period and the mean of times collected provided the time values (minutes) of assembly of each product. As a LP algorithm, the Simplex algorithm was used from Solver® software to achieve the informations to make the best decision-making.

2 Mathematical model of simplex method

The search for process improvements and revenue maximization usually highlights the use of algorithms and mathematical models that help to find a satisfactory result, within the restrictions imposed. Among the methods, it can be highlighted: Genetic Algorithm (Gomes et al., 2018a), Ants Colony (Vuong et al., 2018), Particle Swarm (Liu et al., 2018), SunFlower (Gomes et al., 2018b), among others. However, for linear problems, one must use an algorithm that adequately contemplates the structure of the problem, such as the Simplex Method.

According to De Cosmis and De Leone (2012), it follows the steps for executing an iteration of the Simplex algorithm. In a single iteration for the primal (in Simplex method), one must assume the current basis $B \subseteq \{1, \ldots, n\}$ and $x \in \mathbb{X}$ as the current Basic Feasible Solution (BFS), thus $x_n = A_n^{-1}b \geq 0$, where $xn = 0$ and $|B| = m$. Therefore, $B = \{j_1, j_2, \ldots, j_m\}$ and $N = \{1, \ldots, n\} \setminus B = \{j_{m+1}, \ldots, j_n\}$. From this, it is calculated $\pi = A^{-1}_B c_B$ and the reduced cost vector $\bar{c}_{jk} = c_{jk} - A_{jk}^T \pi$, $k = m + 1, \ldots, n$.

If $\bar{c}_{jk} \geq 0$, $\forall k = m + 1, \ldots, n$, the current point is a BFS, that is, the algorithm is finished. Otherwise, if $\bar{\pi}$ is not greater than or equal to zero, select $j_r$ with $r \in \{m + 1, \ldots, n\}$ such that $\bar{c}_{jr} < 0$. In this way, such a variable can enter the database.

In this case, it is calculated $\bar{A}_{ij_r} = A_B^{-1}A_{ij_r}$. If $\bar{A}_{ij_r} \leq 0$ the problem is unlimited at the bottom, so the algorithm is interrupted.

If not, calculate:

$$\bar{\pi} = \min_{i: \bar{A}_{ij} > 0} \left\{ \left( A_B^{-1}b \right)_i / A_{ij} \right\}.$$ 

Thus, $s \in \{1, \ldots, m\}$ such that

$$\left\{ \left( A_B^{-1}b \right)_i / A_{ij} \right\} = \bar{\pi}; j_s$$

be the variable that comes out.

In view of this, it must define $x_{j_s} = 0, k = m + 1, \ldots, n, k \neq r; x_{j_r} = \bar{\pi}; x_{j_k}(\rho) = A_{j_k} b - \bar{\pi} A_{j_r}$ and

$$B = B \setminus \{j_r\} \cup \{j_k\} = \{j_1, j_2, \ldots, j_{m-1}, j_k, j_{m+1}, \ldots, j_n\}.$$ 

As well, De Cosmis and De Leone (2012) states that when a non-degenerate step is performed, the value of the objective function decreases strictly. In this way, the Simplex method will be completed after a finite number of iterations.
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if all BFS are non-degenerate. If there is degeneracy, the value of the objective function must remain constant and the algorithm will go into infinite loop. In conclusion, specific rules must be implemented to this algorithm to avoid this situation.

3 Case study

According to United Nations Conference on Trade and Development [UNCTAD] (2015), Brazil is among the countries that generate greater revenues in B2C (business-to-customer) - trade conducted directly between the company and the final consumer - considering the number of online buyers. In a real case of an e-commerce company, it was evaluated the behaviour of a particular assembly and packaging sector to increase its revenue. In this sector, five different products are worked. The products have net revenue of: US$ 2,390.90 (P1); US$ 2,290.90 (P2); US$ 2,190.90 (P3); US$ 2,090.90 (P4) and; US$ 990.90 (P5).

According to the company’s sales history, it is known that the product P1 has a daily demand from 3 to 12 units. The product P2 has a minimum demand of 2 daily units and a maximum demand of 12 daily units. For products P3 and P4, the maximum daily demand is, respectively, 7 and 8 units per day. Finally, product P5 has a minimum demand of 2 units and a maximum demand of 11.

Before being dispatched, each product goes through the processes of quality and packaging. An overview of the process without any kind of interference was done before collecting the data and designing a mathematical model that represents the reality of the company. After obtaining this wide view of the process, the data collection was performed randomly for seven days from 10 a.m. to 3 p.m. using filming. The data are available in Table 1. Using a statistical counter, the mean time spent in this process was estimated in minutes; this is illustrated in Figure 1. Therefore, one has:

- $\Phi_{P1} = 10$ minutes;
- $\Phi_{P2} = 9.5$ minutes;
- $\Phi_{P3} = 7$ minutes;
- $\Phi_{P4} = 8.5$ minutes;
- $\Phi_{P5} = 6$ minutes.

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<th>$P_3$</th>
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Figure 1 – Graphical representation of time distributions

4 A linear programming optimization model applied to the decision-making process of a Brazilian e-commerce company

4.1 Application

From the information collected, described in the previous section, it is possible to design the mathematical model for this case study. Adopting the values of the products as constants $\beta_i$, one has the decision variables of the model as the quantity of products to be checked and assembled for sending, $x_1, x_2, ..., x_5$ for products $P_1, P_2, ..., P_5$, respectively.

The products are not divisible, so the values generated for $x_n$ must be integers. Thus, according to the information collected, one can formulate the constraints of the model.

Inequality constraints represent the demand for products, i.e., the daily amount that the organization can meet. As an additional restriction, \( m(x) \), it can be stated that the prepared and
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The Equation 1 presents the objective function of the problem.

\[ \text{Max } F(y) = \sum_{j=0}^{4} \beta_j x_j = 2.390.90 x_1 + 2.290.90 x_2 + 2.190.90 x_3 + 2.090.90 x_4 + 990.90 x_5 \]

Subject to:

\[
\begin{align*}
& g(x) = \sum_{j=0}^{4} \phi_j x_j = 10x_1 + 9.5x_2 + 7x_3 + 8.5x_4 + 6x_5 \leq 300 \\
& h(x) = 3 \leq x_1 \leq 12 \\
& i(x) = 2 \leq x_2 \leq 12 \\
& j(x) = x_3 \leq 7 \\
& k(x) = x_4 \leq 8 \\
& l(x) = 2 \leq x_5 \leq 11 \\
& m(x) = x_n \geq 0; \quad n = \{1, 2, ..., 5\}
\end{align*}
\]

4.2 Results

Through the application of linear programming by the Simplex method, it was reached an optimal solution, generating the values of P1 to P5 aiming to maximize the daily revenue and respecting the restrictions imposed. According to the results found by Solver®, it is possible to verify that all available time (5:00 hour or 300 min) has been used to check and pack the products. Thus, only P3 and P4 products meet every daily demand. Considering the restrictions, it is necessary to prepare and pack the quantity demanded of these products (7 and 8 units, respectively).

Products P1, P2 and P5 meet the minimum restriction of their units. However, the result shows that these products do not meet part of the daily demand in quantities of 5, 2 and 8 units, respectively.

Therefore, the model shows the quantity required to be checked and packaged for each product, in which seven units will be made for P1, ten for product P2, seven for product P3, eight for product P4 and three units for product P5, thus reaching an optimal solution for the model, maximizing revenue by US$ 74,681.50.

4.3 Simulated scenario

From the result found, it is verified that the time restriction was totally used, i.e., it is a scarce resource where any change in it will cause a change in the optimal solution and consequently in the revenue. There are also scarce restrictions on P3 and P4 products, in which every quantity demanded is attended by the company.

The products P1, P2 and P5 in the solution shows that the company no longer meets the market with these three products in 5, 2 and 8 units respectively, even when meeting the minimum demand. For this problem, the company has some solutions such as: application of resources (since the demand for these products is bigger than the company can meet, it would be up to it to use part of the investments in marketing these products for another activity); increase of the market value of these products; increase of the number of employees for this function.
In order to verify this last hypothesis, the simulation of the hypothetical model was performed based on the use of two employees from historical data; therefore, the time of activity has been reduced. The mathematical model can be verified in Equation 2, describing only the constraint \( g(x) \), in which it was altered. The objective function \( F(Y) \) and the other constraints \( [h(x) \text{ to } m(x)] \) remain the same as in Equation 1.

\[
 g (x) = \sum_{j=0}^{\phi_0 x_j} = 5x_1 + 4.75x_2 + 3.5x_3 + 4.25x_4 + 3x_5 \leq 300 \quad (2)
\]

In view of this new analysis, it would be up for the company to assess whether it is feasible to add it to such activity, given the availability and value of the labour of another operator and equipment, since a significant increase in revenue occurred.

5 Conclusion

This paper uses a linear programming model to optimize a decision-making process of a Brazilian company of e-commerce using the Simplex method. The data of the times of each product of the process was collected to represent the reality of the company.

From the mathematical abstraction of the problem, it was possible to apply the optimization method to find the optimal solution of the problem. The result would provide a revenue of US$ 74,681.50, meeting all the constraints of the model, but presenting different clearance values. Before this diagnosis, a hypothetical simulation was made adding another employee, reducing the time restriction.

The new solution showed that the new situation would meet all the demands of products, bringing a new daily revenue of US$ 99,145.00 to the company, with a considerable increase of 32.76%, from the previous revenue.

From this hypothesis, it is possible to verify that all demand can be met, increasing daily revenue by 32.76%, with a value of US$ 99,145.00 per day. In addition, operators would end their activities 91.5 minutes (01:31:30 hour) earlier than the cut-off time, so they could engage in another activity at this time.

Finally, it can be inferred from the results that prioritizing products that generate higher revenues may not be the best decision-making strategy. Moreover, the simulation that search to meet the restriction of clearance shows that the addition of an employee would add bigger profitability to the company. These results are already in application in the company studied. For future applications, the same method can be used for other sectors, besides making a model that contemplates all sectors, realizing experimental designs from established techniques such as Full Factorial Design and the Response Surface Methodology.

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